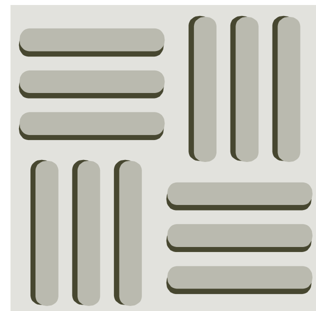
SPECIAL LINES



You may have noticed that most (if not all) of the lines you've seen so far in your course have had either positive or negative *slopes*. You may also have observed that every line had both variables *x* and *y* in its equation. But there are lines whose slopes are neither positive nor negative, and lines whose equations have only one variable in them. This chapter deals with these special lines.

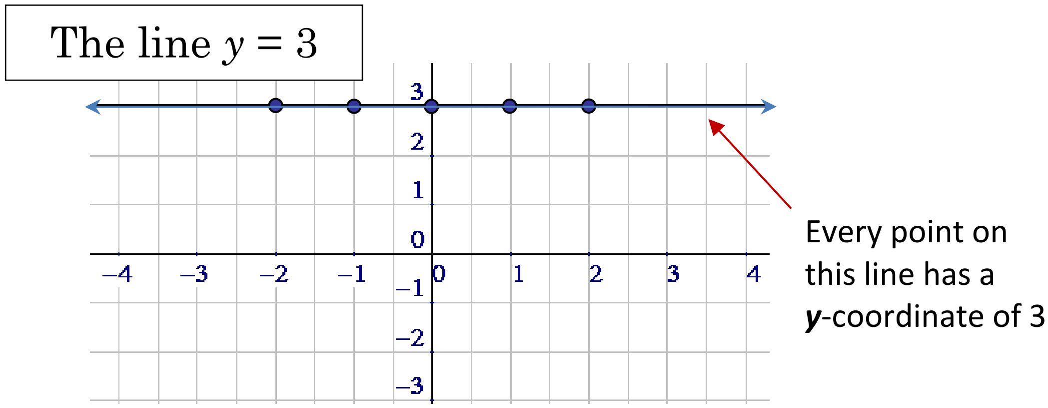
❑ *GRAPHING*

<u>EXAMPLE 1:</u> **Graph the line $y = 3$.**

<u>Solution:</u> This strange little equation doesn't even have an x in it. That's fine — we just think up our favorite x's, and then understand that y is going to be 3 <u>regardless of the x-value we choose</u>. That is, y is a constant; it doesn't depend on x. Here's a possible table of values for this line. [You are more than welcome to choose x-values different from the ones I've chosen, but it won't make any difference in the final graph.]

x	y
–2	3
–1	3
0	3
1	3
2	3

We therefore have the points (–2, 3), (–1, 3), (0, 3), (1, 3), and (2, 3). Plotting these five points, and then connecting them with a straight line, produces the following **horizontal** line; notice that the y-intercept of this line is (0, 3), and that there's no x-intercept.



EXAMPLE 2: **Graph the line $x = -2$.**

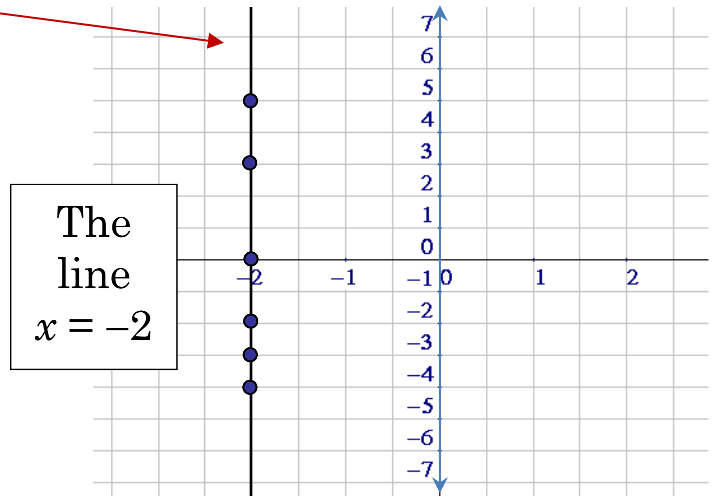
Solution: This one's as goofy as the previous one — but this time the y is missing. But more importantly, the equation clearly informs us that x must be -2. Any other choice of x would contradict this requirement. Moreover, since there's no y in the equation, we can choose any number we'd like for y. This leads to a collection of points like this:

(–2, –4) (–2, –3) (–2, –2) (–2, 0) (–2, 3) (–2, 5)

The key to every point on the line $x = -2$ is that the x-coordinate of the point must be –2, while the y-coordinate can be any number (even numbers like π and $-\sqrt{2}$).

When we plot these points and connect them with a straight line, we get the following **vertical** line; note that the x-intercept of this line is (–2, 0) and that there's no y-intercept.

Every point on this line has an ***x***-coordinate of –2.



Homework

1. Graph each line by plotting at least three points:

   a. $y = 4$ b. $y = -3$ c. $x = 5$

   d. $x = -1$ e. $y = 0$ f. $x = 0$

2. a. The horizontal line $y = 0$ is the ________.

   b. The vertical line $x = 0$ is the ________.

3. a. Is the line $x = 1{,}000{,}000$ is horizontal or vertical?

   b. Is the line $y = \sqrt{1679}/\pi$ is horizontal or vertical?

4. At what point do the lines $x = 17$ and $y = -99$ intersect?

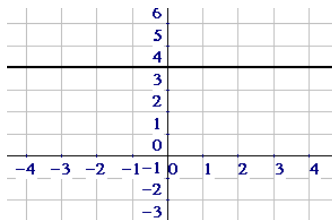
5. Find the intercepts of each line:

   a. $x = 3$ b. $y = -2$ c. $x = 0$ d. $y = 0$

   e. $y = 5$ f. $x = -\pi$ g. $x = \sqrt{2}$ h. $y = x$

❑ THE SLOPE OF A HORIZONTAL LINE

We recall (from Homework 1a) that the graph of the line with the equation $y = 4$ is a horizontal line four units above the x-axis.



Notice that the graph has a y-intercept at (0, 4) but has no x-intercepts. Other points on this horizontal line include (3, 4), (–20, 4), (π, 4), and $(-\sqrt{7}, 4)$. In other words, in the formula $y = 4$, x can be any number, but y must be 4.

Now it's time to calculate the slope of this horizontal line. We need a pair of points on this line — we'll use (3, 4) and (–20, 4).

$$m = \frac{\Delta y}{\Delta x} = \frac{4-4}{3-(-20)} = \frac{0}{23} = \mathbf{0}$$

Since all horizontal lines ought to have the same slope, we can be confident in drawing the following conclusion:

> The slope of any horizontal line is 0.

❑ THE SLOPE OF A VERTICAL LINE

Do you remember what the graph of $x = -2$ looks like? Return to Example 2 and recall that it's a vertical line with x-intercept (–2, 0).

To obtain the slope of this line, we'll use the points (–2, 0) and (–2, 5):

$$m = \frac{\Delta y}{\Delta x} = \frac{5-0}{-2-(-2)} = \frac{5}{-2+2} = \frac{5}{0} = \textbf{Undefined}$$

The conclusion that the slope is "undefined" is based on the fact that division by zero is undefined. We might also observe the "steepness" of the vertical line. It's so steep that no number could measure it, so "undefined" is a good way to describe the slope. Since all vertical lines should have the same slope,

> The slope of any vertical line is *undefined*.

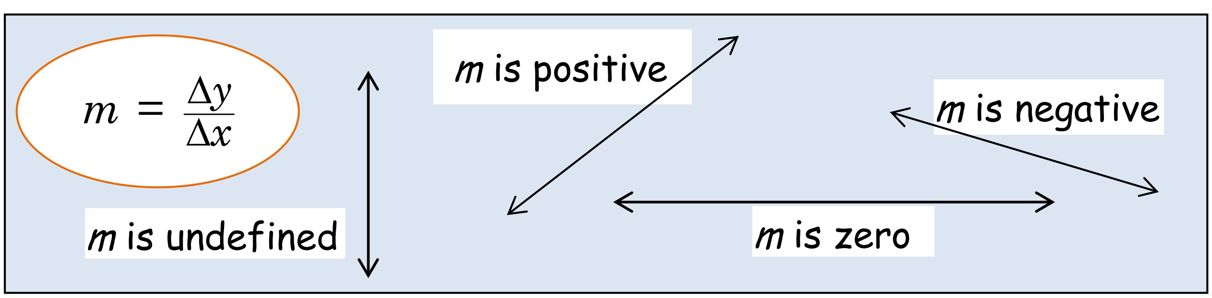
Homework

6. For each line, i) find two points on the line
   ii) use these points and $m = \dfrac{\Delta y}{\Delta x}$ to find its slope

a. $y = 3$ b. $x = 4$ c. $y = -19$

d. $x = -44$ e. $x = 0$ f. $y = 0$

The following diagram is a summary of our notion of ***slope***:



❑ MORE HORIZONTAL AND VERTICAL LINES

We know that a horizontal line always has an equation of the form "y = *some number*," while a vertical line always has an equation of the form "x = *some number*." We've also learned that a horizontal line has a slope of 0, while the slope of a vertical line is undefined. We put all this info into a little table to help us see all the essential facts about horizontal and vertical lines.

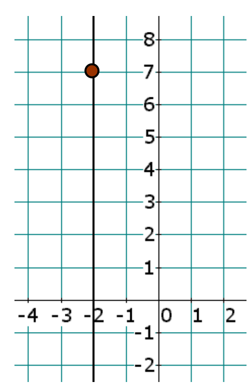
Equation	Type of Line	Slope
y = some number	horizontal	zero
x = some number	vertical	undefined

<u>EXAMPLE 3:</u>

A. **Find the equation of the horizontal line passing through the point (5, 3).**

<u>Solution:</u> A horizontal line has an equation of the form y = *some number*. Since (5, 3) is on the line, the equation of the line must be $\boldsymbol{y = 3}$.

B. **Find the equation of the vertical line passing through the point (–2, 7).**



<u>Solution:</u> A vertical line has an equation of the form x = *some number*. Since (–2, 7) is on the line, the line must have the equation $\boldsymbol{x = -2}$.

C. **Find the equation of the line whose slope is 0 and which passes through the point (–5, 9).**

<u>Solution:</u> If a line has a slope of 0, it must be a horizontal line whose equation must be of the form y = *some number*. Because (–5, 9) lies on the line, the answer is $\boldsymbol{y = 9}$.

D. **A line has an undefined slope and passes through the point (7, –12). What is the equation of the line?**

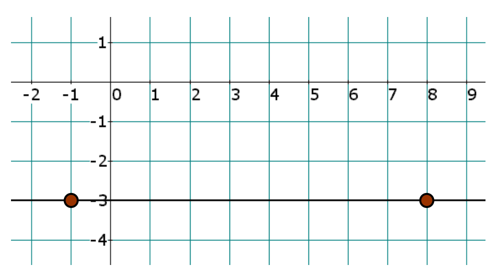
<u>Solution:</u> An undefined slope implies a vertical line, which means an equation like x = *some number*. Since (7, –12) is on the line, its equation must be $\boldsymbol{x = 7}$.

E. **What is the equation of the line passing through the points (9, 5) and (9, –2)?**

<u>Solution:</u> Plot the two points and you'll notice that (9, 5) is directly above (9, –2), yielding a vertical line. The equation must be $\boldsymbol{x = 9}$.

F. **Find the equation of the line passing through (8, –3) and (–1, –3).**

<u>Solution:</u> A quick sketch shows that (8, –3) lies directly to the right of (–1, –3). This creates a horizontal line whose equation is $\boldsymbol{y = -3}$.



Homework

7. Describe the line $y = 17$.

8. Describe the line $x = -99$.

9. Find the line with a slope of 0 and passing through the point (1, 0).

10. The line $y = 8$ is (horizontal, vertical) and its slope is ______.

11. What is the equation of the line passing through (3, 11) and (–3, 11)?

12. Find the equation of the horizontal line passing through the point (–13, 17).

13. Find the line with a slope of 0 and passing through the point (–1, 7).

14. The line $y = -6$ is (horizontal, vertical) and its slope is ______.

15. What is the equation of the line passing through (–17, –13) and (7, –13)?

16. Find the equation of the vertical line passing through the point (–4, –9).

17. Find the line with a slope of 0 and passing through the point (2, 1).

18. The line $x = -4$ is (horizontal, vertical) and its slope is ______.

19. What is the equation of the line passing through (–1, 9) and (–1, 11)?

20. Find the equation of the horizontal line passing through the point (–1, 0).

21. Find the line with an undefined slope and passing through the point (3, 7).

22. The line $x = -8$ is (horizontal, vertical) and its slope is ______.

23. What is the equation of the line passing through (–5, 10) and (–5, 6)?

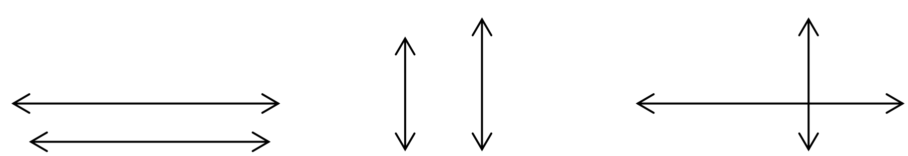
24. Find the equation of the vertical line passing through the point (–18, –11).

25. Find the line with a slope of 0 and passing through the point (–5, –6).

❑ PARALLEL AND PERPENDICULAR LINES

Would you agree that two different vertical lines never intersect? When two lines (in the same plane) never intersect, we say they're ***parallel***. So, for example, the lines $x = 3$ and $x = -4$ are parallel, since each is vertical. Now consider a pair of different horizontal lines. Clearly, they're parallel, too. Thus, for example, the lines $y = 2$ and $y = \pi$ are also parallel.

Now consider a vertical line and a horizontal line. They must meet at a 90° angle, and we say that the two lines are ***perpendicular*** (in the same way that the two legs of a right triangle are perpendicular to each other). We can therefore say that the lines $x = 5$ (vertical) and $y = -3$ (horizontal) are perpendicular.



Parallel lines Parallel lines Perpendicular Lines

Parallel lines and perpendicular lines can also be at an angle; you'll see how that works in the chapter *Parallel and Perpendicular Lines*.

EXAMPLE 4:

A. **Find the equation of the line which is parallel to the line $x = 7$ and which passes through the point (5, 3).**

Solution: The line $x = 7$ is vertical. Any line parallel to this line must also be vertical. What vertical line passes through the point (5, 3)? The line $\boldsymbol{x = 5}$ does.

B. **Find the equation of the line which is perpendicular to the line $x = -5$ and which passes through the point (–2, –9).**

Solution: Since the line $x = -5$ is vertical, the perpendicular line we're seeking has to be horizontal. What is the equation of the horizontal line passing through (–2, –9). The answer is $\boldsymbol{y = -9}$.

C. **Find the equation of the line which is parallel to the line $y = 17$ and which passes through the point (–1, 3).**

Solution: This time the given line $y = 17$ is horizontal, and since we seek a parallel line, it also must be horizontal. And the horizontal line passing through the point (–1, 3) is certainly $\boldsymbol{y = 3}$.

D. **Find the equation of the line which is perpendicular to the line $y = 11$ and which passes through the point (6, –3).**

Solution: The line $y = 11$ is horizontal, so we need a vertical line passing through (6, –3). That line is $\boldsymbol{x = 6}$.

Homework

26. Fill in each blank with either the word 'parallel' or 'perpendicular':

    a. Two different vertical lines are _____.
    b. Two different horizontal lines are _____.
    c. A vertical line and a horizontal line are _____.

27. Fill in each blank with either the word 'vertical' or 'horizontal':

    a. A line which is parallel to a vertical line must be ___.
    b. A line which is perpendicular to a horizontal line must be ___.
    c. A line which is parallel to a horizontal line must be ___.
    d. A line which is perpendicular to a vertical line must be ___.

28. a. Are the lines $x = 9$ and $x = -1$ parallel or perpendicular?
    b. Are the lines $y = 7$ and $y = 0$ parallel or perpendicular?
    c. Are the lines $x = -9$ and $y = 7$ parallel or perpendicular?

29. a. Give an example of a line which is parallel to $x = 5$.
    b. Give an example of a line which is parallel to $y = -4$.
    c. Give an example of a line which is perpendicular to $y = -4$.
    d. Give an example of a line which is perpendicular to $x = 8$.

30. Fill in each blank with either the word 'vertical' or 'horizontal':

    a. A line which is parallel to the line $y = 7$ must be _____.
    b. A line which is perpendicular to the line $x = 3$ must be _____.
    c. A line which is parallel to the line $x = 8$ must be _____.
    d. A line which is perpendicular to the line $y = -3$ must be _____.

31. a. Find the equation of the line which is parallel to the line $x = 9$ and which passes through the point (1, 7).

b. Find the equation of the line which is perpendicular to the line $x = -3$ and which passes through the point (–7, 0).

c. Find the equation of the line which is parallel to the line $y = 10$ and which passes through the point (–5, 8).

d. Find the equation of the line which is perpendicular to the line $y = -9$ and which passes through the point (7, –2).

Review Problems

32. a. Graph the line $x = -3$ by plotting three points.

b. Is the line horizontal or vertical?

c. Find all the intercepts of the line.

d. Use two of the points and $m = \frac{\Delta y}{\Delta x}$ to calculate the slope.

33. a. Graph the line $y = 2$ by plotting three points.

b. Is the line horizontal or vertical?

c. Find all the intercepts of the line.

d. Use two of the points and $m = \frac{\Delta y}{\Delta x}$ to calculate the slope.

34. a. What is the equation of the x-axis?

b. What is the equation of the y-axis?

35. The line $y = \sqrt{3}$ is (horizontal, vertical) and its slope is ______.

36. The line $x = \sqrt{2\pi}$ is (horizontal, vertical) and its slope is ______.

37. Graph the line $y = x$. Is it horizontal, vertical, or diagonal? What is its slope?

38. Find the equation of the horizontal line passing through the point (17, 99).

39. Find the equation of the vertical line passing through the point (34, −44).

40. Find the equation of the line with undefined slope passing through the point (2, −π).

41. Find the equation of the line with 0 slope passing through the point (2, −π).

42. What is the equation of the line passing through (2, 7) and (2, 1)?

43. What is the equation of the line passing through (1, 7) and (0, 7)?

44. Find the equation of the line which is parallel to the line $x = 14$ and which passes through the point (−2, −9).

45. Find the equation of the line which is perpendicular to the line $y = -23$ and which passes through the point (π, 0).

46. True/False:

a. The line $y = \sqrt{2}$ is horizontal.

b. The line $x = 3$ has an undefined slope.

c. The line $y = 5$ has exactly one intercept.

d. The vertical line passing through (2, 7) is $x = 7$.

e. The equation of the x-axis is $y = 0$.

f. The line $x = -1$ has infinitely many intercepts.

g. The point (7, 9) lies on the line $y = 9$.

h. The line $x = -8$ has a negative slope.

i. The slope of the line $y = 3x + 4$ is 3.

j. The line passing through (3, π) and (3, 1) is $x = 3$.

k. The line $y = x$ is horizontal.

l. A line can have two intercepts.

m. The point (−2, 5) lies on the line $x = 5$.

n. The line $y = 7$ has an undefined slope.

o. All horizontal lines have the same slope.

p. The equation of the y-axis is $y = 0$.

q. The line passing through (1, 2) and (1, 0) is $y = 1$.

r. A line can have exactly one intercept.

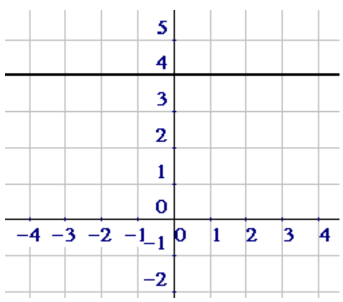
s. A line can have infinitely many intercepts.

t. The lines $x = 3$ and $y = 4$ are parallel.

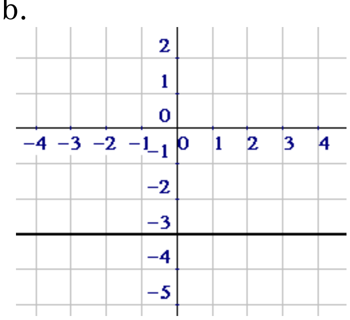
u. The slope of the line $y = x$ is 1.

Solutions

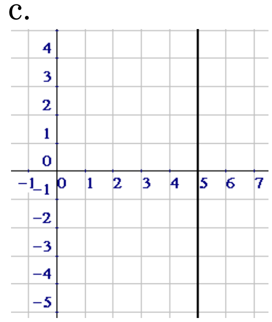
1. a.



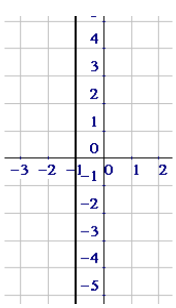
b.



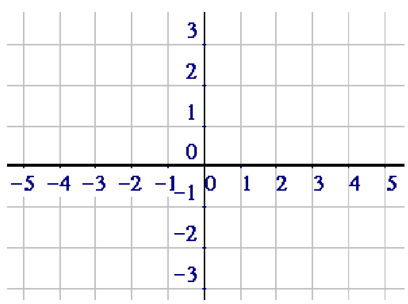
c.



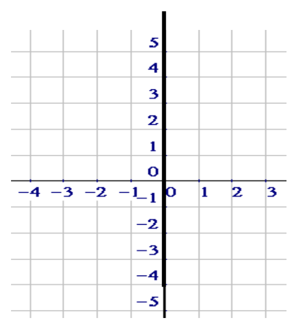
d.



e.



f.



2. a. x-axis b. y-axis **3.** a. vertical b. horizontal **4.** (17, −99)

5. a. x-intercept: (3, 0); No y-intercept

b. No x-intercept; y-intercept: (0, –2)

c. x-int: (0, 0); y-int: all the points on the y-axis are y-intercepts

d. x-int: all the points on the x-axis are x-intercepts; y-int: (0, 0)

e. No x-intercept; y-intercept: (0, 5)

f. x-intercept: $(-\pi, 0)$; No y-intercept

g. x-intercept: $(\sqrt{2}, 0)$; No y-intercept

h. x-int: (0, 0); y-int: (0, 0)

6. a. e.g., (2, 3) and (–4, 3); $m = \frac{3-3}{2-(-4)} = \frac{0}{6} = 0$

b. e.g., (4, 7) and $(4, \pi)$; $m = \frac{7-\pi}{4-4} = \frac{7-\pi}{0} =$ Undefined

c. $m = 0$ d. $m =$ Undefined

e. $m =$ Undefined f. $m = 0$

7. $y = 17$ is a horizontal line 17 units above the x-axis. Its y-intercept is (0, 17), but it has no x-intercepts; its slope is 0.

8. $x = -99$ is a vertical line 99 units to the left of the y-axis. Its x-intercept is (–99, 0), but it has no y-intercepts; its slope is undefined.

9. $y = 0$ **10**. horizontal; 0 **11**. $y = 11$ **12**. $y = 17$ **13**. $y = 7$

14. horizontal; 0 **15**. $y = -13$ **16**. $x = -4$ **17**. $y = 1$

18. vertical; undefined **19**. $x = -1$ **20**. $y = 0$ **21**. $x = 3$

22. vertical; undefined **23**. $x = -5$ **24**. $x = -18$ **25**. $y = -6$

26. a. parallel b. parallel c. perpendicular

27. a. vertical b. vertical c. horizontal d. horizontal

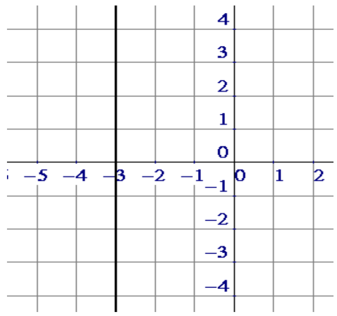
28. a. parallel b. parallel c. perpendicular

29. a. $x = 23$, for example; any line of the form $x =$ *some number* would work.
b. $y = 9$, for example; any line of the form $y =$ *some number* would work.
c. $x = -\pi$ for example; any line of the form $x =$ *some number* would work.
d. $y = -3$, for example; any line of the form $y =$ *some number* would work.

30. a. horizontal b. horizontal c. vertical d. vertical

31. a. $x = 1$ b. $y = 0$ c. $y = 8$ d. $x = 7$

32. a. For instance, (–3, 0), (–3, –2), and (–3, 4) are three points on the line. Plotting these points and connecting them produces the graph:



b. The line is vertical.

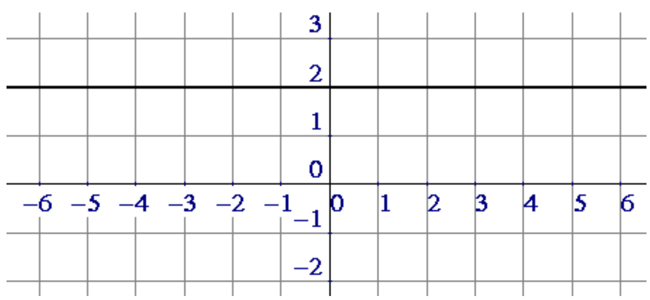
c. The only intercept of this line is (–3, 0).

d. Using the first two of the three points, we find the slope:

$$m = \frac{\Delta y}{\Delta x} = \frac{0-(-2)}{-3-(-3)} = \frac{0+2}{-3+3} = \frac{2}{0},$$

and therefore the slope is undefined.

33. a. For example, (–1, 2), (3, 2), and (4, 2) are three points on the line. Plotting these points and connecting them produces the graph:



b. The line is horizontal.

c. The only intercept of this line is (0, 2).

d. Using the first two of the three points, we find the slope:

$$m = \frac{\Delta y}{\Delta x} = \frac{2-2}{-1-3} = \frac{0}{-4} = 0$$

and therefore <u>the slope is 0</u>.

34. a. $y = 0$ b. $x = 0$

35. horizontal; 0

36. vertical; undefined

37. diagonal; 1

38. $y = 99$

39. $x = 34$

40. $x = 2$

41. $y = -\pi$

42. $x = 2$

43. $y = 7$

44. $x = -2$

45. $x = \pi$

46. a. T b. T c. T d. F e. T f. F g. T h. F i. T
j. T k. F l. T m. F n. F o. T p. F q. F r. T
s. T t. F u. T

"This thing we call 'failure' is not the falling down . . .
but the staying down."

Mary Pickford